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RESPONSE TO ROBERTA FERRARIO

In the first two sections of her commentary on my work, Ferrario gives an excellent summary of the views expressed in my recent book [2002], especially on the three critical concepts of axiomatization, representation, and invariance. I also like her comments in the last part of section 2, namely, section 2.3, where she stresses two points that she states about my position that are fairly unusual. The first one is the heuristic value that in earlier publications I have assigned to the axiomatic method. She is right in this because it is a common position to compare heuristics to axiomatic methods, in the sense there is no claim that heuristic methods give a full formal account of the content of a subject, or that axiomatic methods are heuristic. I also want to emphasize another point that she does. This is that I think it is important to distinguish axioms that are heuristically valuable, or put another way, are intuitive in their presentation of content. Some axioms that seem necessary and are very useful in subjects are often described as technical. This usually means they are complicated, difficult to read, and do not have a straightforward intuitive meaning. It is a desirable distinction to be made about axioms that brings the concept of heuristics within the framework of the axiomatic method itself. In my 1983 article on this subject that she cites, the main example of axiomatization I criticized on this score is Mackey's axiomatization of quantum mechanics ([1957],

Epistemologia XXIX (2006), pp. 385-388.

[1963]). The non-heuristic character of these axioms is I think the main reason they are seldom mentioned by physicists.

The second unusual point that Ferrario mentions is my stress on the use of axiomatic methods in the empirical sciences. I think she is still right about this point, even though by now the use of the axiomatic method, in such subjects as economics, is the natural way to treat a subject in such prominent journals as *Econometrica*. She mentions a point that I have not emphasized myself enough. This is the paradoxical fact that some of the most extensive axiomatizations in use in the social sciences are to be found in the area of measurement, because of the desire to make the procedures of measurement in these new sciences completely explicit. Here the axiomatic method is used to a fare-the-well, – to give an egocentric reference – as may be found in the three volume treatise *Foundations of Measurement* ([1971], [1989], [1990]) of which I am a co-author. Put another way, the axiomatic method has very much proved itself to be of use not only in empirical sciences but also in the general *methodology* of the empirical sciences.

The remainder of Ferrario's paper is about formal ontologies with a use of axiomatic methods within the framework of first-order logic that has become increasingly popular in computer science. I generally agree with the comparisons she draws between the use of axiomatic methods in the sciences and in formal ontologies. The description of the use of axioms in formal ontologies in section 3.3 of Ferrario's commentary has an outlook and a viewpoint that very naturally goes back to the first modern work on the foundations of the axiomatic method, namely, the 1882 treatise on geometry of Moritz Pasch, which made explicit the formal nature of modern axiomatics, namely, that no intuitive content of any particular interpretation is made a direct part of the axioms. This methodology, well-consolidated by Hilbert in his *Foundations of Geometry* [1899], is now very much the modern view of what one means by the axiomatic method in either mathematics or the sciences. What is said in formal ontologies has direct resonance with the interesting and simple examples given by Pasch in his early work. A point that is more modern than Pasch, and clearly is of great importance in formal ontologies, is the non-categorical character of the axioms (a set of axioms is categorical if and only if any two models of the axioms are isomorphic). The many successful formal axiomatizations of geometry at the end of the nineteenth century and beginning of the twentieth century were mainly aimed at categorical axioms, for example, for Euclidian, hyperbolic, and elliptic geometry. Even though, it should be noticed, non-categorical axioms were familiar in projective geometry, for example, Fano's miniature projective plane of just seven points and seven lines, the report of which was published in 1892, was just one example of the many hundreds of finite geometries that have been considered since. Such finite geometries constitute, in many ways, developments parallel to those considered in formal ontologies.

Finally, I am happy to remark that the approach to formal ontologies shares a pluralism I much advocate in the analysis of structures and theories in the empirical sciences.

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