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## RESPONSE TO MASSIMILIANO BADINO

Badino has divided his comments into four sections. The first is on a fundamental dualism with which I agree, but had not formulated the way he does in thinking about my own work. The dualism is that between invariance and pluralism. The contrast that Badino has in mind is the following. In axiomatizing a theory, for example probability as the main example he uses and one of the most important ones in my own work, we search for the invariant properties to be found in all, or almost all, applications of probability. It is then just the formal properties that have this invariance that are considered in the axiomatization. On the other hand, the applications of a theory of any power and usefulness are manifold and pluralistic in character. In spite of my emphasis in my book *Representation and Invariance of Scientific Structures* [2002] on the concepts of invariance and pluralism, I did not sufficiently emphasize this particular way of bringing the two together. I like what Badino has done. It clarifies the proper role of invariance and that of pluralism in thinking about scientific theories and their empirical content.

The important role that an axiomatic foundation has for the theory of probability and the possible clarification thereby of the notion of invariance, was well recognized by Kolmogorov in his short monograph on the foundations of probability [1933/1950], which was very effective in setting a modern standard for putting

probability in its proper place in the modern set theoretical formulations of mathematics. Here is a quotation from Kolmogorov's preface:

The purpose of this monograph is to give an axiomatic foundation for the theory of probability. The author set himself the task of putting in their natural place, among the general notions of modern mathematics, the basic concepts of probability theory – concepts which until recently were considered to be quite peculiar.

This task would have been a rather hopeless one before the introduction of Lebesgue's theories of measure and integration. However, after Lebesgue's publication of his investigations, the analogies between measure of a set and probability of an event, and between integral of a function and mathematical expectation of a random variable, became apparent. These analogies allowed of further extensions; thus, for example, various properties of independent random variables were seen to be in complete analogy with the corresponding properties of orthogonal functions. But if probability theory was to be based on the above analogies, it still was necessary to make the theories of measure and integration independent of the geometric elements which were in the foreground with Lebesgue. This has been done by Fréchet.

While a conception of probability theory based on the above general viewpoints has been current for some time among certain mathematicians, there was lacking a complete exposition of the whole system, free of extraneous complications. (Kolmogorov [1933/35], p. v)

Pluralism, on the other hand, enters not only in the applications but in the various interpretations or representations of probability that have been advanced in the literature. I survey in some detail these various representations of probability, ranging from the logical to the subjective, in my book [2002]. In this case, the pluralism is not simply one just of application but also of more detailed thinking about probability. Exactly how is it being used in various contexts and by various authors. Because I very much agree with Badino on his main point, I only want to mention one other aspect of my own views. In the closing pages of the long Chapter 5 of my book on representations of probability, I emphasize, finally at the end, a pragmatic attitude. Here pragmatism means that we should accept without rancor the many different ways in which probability has been

interpreted over the last several centuries while it was being developed for applications in many different domains of thought and activity. The pragmatism comes from a realization that complex and deep applications often do not really work well if forced into one unique way of interpreting probability, for example, as a propensity or as a subjective belief. This idea of pragmatism is not based just on my own ruminations, but is illustrated by what has been said about probability by many physicists and also by one of the twentieth-century statisticians best known for a wide range of applications, namely, Frederick Mosteller (Mosteller and Wallace [1964/1984]). In section two of Badino's commentary he focuses on the axiomatization of Kolmogorov that I have just been discussing. I won't say more here, for I almost entirely agree with what he has to say.

His third section concentrates on the propensity theory of probability, to which I devote a long section in Chapter 5 of my book. Here he and I agree on the necessity of having serious substantive mathematical analysis of the properties that any proper theory of propensity must have, in order to qualify as a genuine and important interpretation of probability. As I indicated, earlier in my book, I find that much of the discussion of propensity is lacking in the kind of technical detail needed to make it a serious competitor of relative-frequency or subjective theories of probability.

There is an important point that Badino comments on that I have stressed earlier, but think it needs stressing once again. It is easy to refer in casual conversations about the foundations of probability to "the" propensity interpretation as I did above. In fact, the propensity interpretation or representation is a grammatical mistake, literally, for we should be speaking of propensity representations. Because it seems certainly evident enough that different physical cases of propensity will lead to different structural axioms, not only to relate in a detailed way the physical structures and processes considered to the axioms of probability, but in even more detail to fix the exact nature of the particular probability distributions arising in a given case. So, for example, in the case of radioactive decay, we are led naturally by the physical phenomena to the exponential distribution, but in consideration of the standard theory of errors of meas-

urement in classical physics we are led to the normal distribution (usually called in physics the *Gaussian* distribution).

The fourth section of Badino's commentary is on the subjective interpretation of probability. Again, I agree with much that he has to say about subjective probability. I just want to emphasize one point that I have not stated in writing as much as I have in lectures and conversations. Badino points out my emphasis on having for the subjective view of probability a criterion of "pure rationality", which means as few purely structural axioms as possible. In my book I review various structural axioms and how they restrict the concept of rationality. Here I want to make a skeptical remark about pure theories of rationality. In almost every domain of experience where we have sought for something really simple and pure, the results have ended up being much more complicated. The necessity of structural axioms in the subjective theory of probability in order to have an exact numerical representation, has an analog in a more significant case, the search for simple foundations of mathematics. In the naive and glorious days of the beginning with Frege and Russell, it was thought that something rather straightforward could be found. If anything has turned out to be complicated, it is what is the right axiomatization for the foundations of mathematics from whatever viewpoint one starts: classical, intuitionistic, or even more restricted recursive fragments of arithmetic. Exactly what the axioms should be has no unique satisfactory answer. The philosophical importance of this absence of a completely unique satisfactory answer to almost every fundamental question of this kind that we examine is of much greater philosophical importance, in many ways, than the survey of what particular choices have been the popular ones.

The fifth and final section of Badino's comments are on my probabilistic empiricism. Early in his comments he quotes a remark from my earlier book [1984] on not only the chaotic character of particular physical phenomenon but the general character of all our knowledge of the world. This remark amplifies the one I just made in the preceding paragraph about the problems of having simple axioms of rationality. For example, for subjective probability we also have a problem of finding really solid grounds anywhere but, and I

stress here as part of my firm convictions about science and philosophy, it is naive to expect anything else. In fact, I would hope that in the twenty-first century we will learn something from the enormous range of ideas that were proposed for settling all kinds of fundamental problems but in a naïve way in the twentieth century. What we should have learned is that settling fundamental problems in some fundamental way is a mistake. The right view is to recognize that we will continually be making adjustments and changes as particular situations arise in the future. There is no hope of having one final all-encompassing correct view of any major area of thought or activity. Driving home this point of intellectual modesty about what we can accomplish continues to be a main theme of my own philosophical viewpoint on almost any subject to be mentioned.

Finally, regarding my own general claims about probabilistic metaphysics and Badino's skepticism that I have made always the right distinction, I remain firm in my convictions. To me probabilistic *metaphysics* is itself an empirical enterprise. It is what I call the genuine Aristotelian one. Aristotle's metaphysics is a very different intellectual enterprise from, say, Kant's *Critique of Pure Reason* or the fantasies of the philosophers whom Kant was criticizing. Aristotle's metaphysics is a very general account of many things learned from experience. He makes no attempt to separate experience from metaphysics, and so it is with me. The only way in which metaphysics can be a serious subject is as one that reaches for principles to be found in much, if not all, of experience. Representations and invariances of various kinds are grounded within such a framework by there being partial mathematical results for various theories expressed formally and with the invariance of particular representations, embodied in appropriate theorems. These theorems will never be universal ones in the sense of something holding for experience of a great variety of kinds, but theorems dealing with particular parts of experience, as we expect especially in particular parts of mathematics or particular parts of a branch of science. So another tension, one that I mention as ever present in the kind of enterprise I have embarked on myself and encourage in others, is the tension between the solidity of particular formal results, as expressed in theorems on

representations or invariances, and on the other hand, the particular character of each of these theorems. They are never theorems of some universal metaphysic, but theorems about some particular aspect of the world or of our experience of it. Moreover, the foundational framework within which the theorems are expressed can also be regarded as something that is not grounded once and for all, and for eternity, as was once hoped would be the case with the foundations of mathematics. But will constitute an active and changing enterprise, with new results and new viewpoints as well, as is certainly characteristic of foundational work over the past several decades.

#### REFERENCES

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