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ON THE ORIGINS OF METAMATHEMATICAL INVESTIGATIONS:
INDEPENDENCE AND CONSISTENCY AT THE TURN
OF THE 19TH CENTURY

Abstract

In the last decade of the 19th century Peano and his school offered very important contributions to the abstract view of axiomatic theories and related metatheoretical problems, typically represented by independence and consistency of axioms. Can it be asserted that they really anticipated Hilbert on this subject-matter? The answer could be: obviously yes and of course no! The yes-part concerns the attitude toward primitive concepts, the very notion of hypothetical-deductive system (due to Pieri), the treatment of the independence of axioms and of primitive concepts too (Padoa's method), some investigations by Pieri on the consistency (which was intended as independence of the negation of every axiom from the others). But independence (and consistency) were to be proved through the exhibition of a suitable model, which was the only method conceived by Peano and his school to show the unprovability of a proposition. This leads eventually to the no-part of the answer. Actually Hilbert envisaged a new method, which was completely lacking in the Italian school, that is, to proceed by mathematical induction on theorems in a formalized theory. In spite of the initial criticism by Poincaré, this finally led, in the twenties, to the mature formulation of Hilbert's program.