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ON THE NEGATION OF THE AXIOM OF FOUNDATION

Abstract

The *axiom of foundation* is a principle asserting the non existence, in the domain of set theory, of *non-well-founded* sets, of the form $\dots x_n \in x_{n-1} \in \dots \in x_1 \in x_0$, and, as a particular case, of *circular* sets of the kind $x \in x$ and $x_0 \in x_1 \in \dots \in x_1 \in x_0$.

In a previous article (published in *Epistemologia* XXVIII) we have shown how the major part of mathematicians that contributed to the creation of axiomatic set theory accepted the axiom of foundation and considered non-well-founded sets as theoretical objects without any mathematical interest.

Exceptions to this point of view are the investigations of D. Mirimanoff (1917-20) and P. Finsler (1926), which contain deep mathematical intuitions about non-well-founded-sets. In their works various definitions of isomorphism and equality between sets are analyzed, and the expressive limits of Zermelo's axiom of extensionality in the case of non-well-founded sets are criticized. While considering this kind of questions, Mirimanoff gives a definition of isomorphism that is very similar to the modern definition of *bisimulation*, and Finsler bases the analysis of the concept of set on the discovery of a deep correlation between sets and *graphs*, both fundamental topics of investigation in set theory during the last twenty years.