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THE PROBLEM OF FOUNDATION OF THE MATHEMATICS
IN EARLY WORKS BY BERNARD BOLZANO

Abstract

The main aim of this article is to provide a conceptual summary of the theory of Bernhard Bolzano related to the foundations of mathematics preceding his two great encyclopedic works: the *Grössenlehre* and the *Wissenschaftslehre* (both completed in 1837, even if the former was not published while Bolzano was still alive).

In particular the article tries to uphold two theories:

I) Bolzano believed in the existence of a basis concept as regards all mathematical notions and, in particular, in relation to the concepts of number and quantity: the *Ganze* ("the whole"). The *Ganze* has a dual meaning: 1) it is a *more original* entity if related to numbers and quantities, meaning that they can be defined starting from the *Ganze*; 2) it is *more general* since some branches of mathematics can work without the notions of number or quantity, though the idea that, given some mathematical entities, their set must exist as well, is still indispensable. The *Ganze*, therefore, is a form that pervades every branch of mathematics and is filled each time with different content.

The article, moreover, will explain that Bolzano's formalism should not be confused with Hilbert's, which is typical of the end of the nineteenth and beginning of the twentieth century.

II) The logic implied in the early works by Bolzano cannot be considered either extensional or intensional; rather, he tries to base his own speculations on such concepts as the *Ganze*, where intensional and extensional meanings coexist. Any *Ganze*, once the elements which constitute it have been determined, can, in fact, be treated as a mere aggregate of objects, and the way by means of which a 'whole' is achieved, however, must necessarily make reference to a certain compository law, to an intention.

Finally a hint to the sources: I have consulted Bolzano's manuscripts (dating to the period 1809-1820) found in Bolzanian *Opera omnia*, published by Friedrich Fromann Verlag, Stuttgart-Bad Cannstatt.